A Novel Identification Method of Thermal Resistances of Thermoelectric Modules Combining Electrical Characterization Under Constant Temperature and Heat Flow Conditions

Saima Siouane, Slaviša Jovanović and Philippe Poure

Abstract—The efficiency of a Thermoelectric Module (TEM) is not only influenced by the material properties, but also by the heat losses due to the internal and contact thermal resistances. In the literature, the material properties are mostly discussed, mainly to increase the well-known thermoelectric figure of merit ZT. Nevertheless, when a TEM is considered, the separate characterization of the materials of the p and n elements is not enough to have a suitable TEM electrical model and evaluate more precisely its efficiency. Only a few recent papers deal with thermal resistances and their influence on the TEM efficiency; mostly, the minimization of these resistances is recommended, without giving a way to determine their values. The aim of the present paper is to identify the internal and contact thermal resistances of a TEM by electrical characterization. Depending on the applications, the TEM can be used either under constant temperature gradient or constant heat flow conditions. The proposed identification approach is based on the theoretical electrical modeling of the TEM, in both conditions. It is simple to implement, because it is based only on open circuit test conditions. A single electrical measurement under both conditions (constant-temperature and constant-heat) is needed. Based on the theoretical electrical models, one can identify the internal and thermal resistances.

Index Terms—Thermoelectric Module (TEM); thermal resistances; contact thermal resistance; electrical characterization; identification; electrical model.

I. INTRODUCTION

FOR a small scale power production, a Thermoelectric Module (TEM) appears as the most promising solution. It has the advantages of no moving parts, high reliability, long lifetime being noiseless and environmentally friendly [1], [2]. A thermoelectric module is a solid state semiconductor device that converts heat into electricity by using the Seebeck effect. The TEM is made of p and n type semiconductor elements that are electrically connected in series, to generate a higher voltage, and thermally in parallel to keep each semiconductor element at the same thermal gradient. These elements are typically arranged in a planar array and sandwiched between two thermal ceramic plates, where heat is transferred between the hot and the cold sides [3]. Each junction formed by the p and n elements is connected to metal contacts (see Fig. 1). When an input heat flow $Q_H$ is applied at the hot side, a temperature gradient is created between the hot and cold sides of the TEM, and therefore the appearance of an electrical current $I$ is observed. This phenomenon is known as ‘Seebeck effect’.

In Fig. 1 $T'_H$ and $T'_C$ denote the external hot and cold sides temperatures applied to the TEM respectively, whereas $T_H$ and $T_C$ denotes the internal hot and cold sides temperatures applied to the p and n elements. Notice that the thermal gradient $\Delta T = T_H - T_C$ applied externally to the TEM is lower than the thermal gradient $\Delta T' = T'_H - T'_C$ across the p-n junction elements. This is due to the contact thermal resistance $\theta_c$ between the TEM’s semiconductors and their metal contacts. This contact thermal resistance is considered in the TEM electrical modeling presented in this paper.

Potential applications of thermoelectric modules are in thermal energy harvesting to power wireless sensors or microelectronic devices such as wearable medical sensors [4]. They can also be an energy source to power wristwatches [5]. Moreover, TEMs can also be combined with photovoltaic cells by first converting the solar energy into electricity and heat, and then using the wasted heat to generate electricity [6].

A thermoelectric module is subjected to a limit on its thermoelectric conversion efficiency ($\eta$), typically around 5% [7]. As a result, the applications of the thermoelectric module have been limited to specialized domains where the cost
The thermoelectric conversion efficiency is defined as the ratio between the energy supplied to the load $P$ and the heat energy $Q_H$ absorbed at the hot junction of a TEM:

$$\eta = \frac{P}{Q_H}$$

(1)

The enhancement of the thermoelectric module efficiency is related to the material properties [9]. The key to expanding the range of applications of thermoelectric modules is the development of more efficient materials by improving their internal physical properties [8]. The thermoelectric module efficiency is evaluated by the well-known figure of merit $ZT$ defined as:

$$ZT = \frac{\alpha^2 \theta_m T}{R_E T}$$

(2)

where $\alpha$ is the Seebeck coefficient, $\theta_m$ the internal thermal resistance, $R_E$ the electrical resistance of the TEM, and $T$ the average temperature given by:

$$T = \frac{T_H + T_C}{2}$$

(3)

A greater figure of merit $ZT$ indicates more efficient thermoelectric materials [10] having a higher Seebeck coefficient, a lower electrical resistance despite of a higher internal thermal resistance.

Different thermoelectric materials have different values of figure of merit which limit their use to applications with specific temperature range. Fig. 2 [11] shows that for lower temperatures (200 to 400K) bismuth telluride ($\text{Bi}_2\text{Te}_3$) is preferred while for temperatures between 600 and 800K the material lead telluride ($\text{PbTe}$) is recommended. At higher temperatures (from 800 to 1300K) silicon germanium ($\text{SiGe}$) is used [12], [13]. For all these materials and temperature ranges it holds that $ZT$ is around $1$ [14], [15].

**II. TEM ELECTRICAL MODELING**

The electro-thermal model of a TEM is given in Fig. 3. The electrical part of the model is drawn in black whereas the thermal one is shown in purple. The contact thermal resistance can be modeled by two series resistors $\theta_c$ at the top and bottom of the TEM. The internal thermal resistance is called $\theta_m$.

![Electro-thermal model of a TEM](image)

A detailed view of the parameters used in the presented TEM model is shown in Table I. In this section, the equivalent electrical model of a TEM under constant temperature gradient conditions is first described. Next, considering constant heat flow conditions, the electrical modeling of a TEM is also presented.
TABLE I
PARAMETERS OF THE TEM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_E)</td>
<td>TEM electrical resistance</td>
</tr>
<tr>
<td>(\theta_m)</td>
<td>TEM internal thermal resistance</td>
</tr>
<tr>
<td>(R_L)</td>
<td>Load resistance</td>
</tr>
<tr>
<td>(\theta_c)</td>
<td>Contact thermal resistance</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Seebeck coefficient</td>
</tr>
<tr>
<td>(T_H)</td>
<td>Temperature applied at the hot side of the TEM</td>
</tr>
<tr>
<td>(T_C)</td>
<td>Temperature applied at the cold side the TEM</td>
</tr>
<tr>
<td>(T_H')</td>
<td>Hot side temperature at the terminals of p-n elements</td>
</tr>
<tr>
<td>(T_C')</td>
<td>Cold side temperature at the terminals of p-n elements</td>
</tr>
<tr>
<td>(Q_H)</td>
<td>Input heat flow</td>
</tr>
<tr>
<td>(Q_C)</td>
<td>Output heat flow</td>
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</tbody>
</table>

A. TEM modeling under constant temperature gradient conditions

In this case, the external thermal gradient \(\Delta T' = T_H' - T_C'\) across the thermoelectric module is maintained constant. Due to the contact thermal resistance \(\theta_c\), the thermal gradient \(\Delta T\) at the terminals of the p and n elements is decreased.

The heat flow rates through the hot side of the TEM (the input heat flow \(Q_H\)) and its cold side (the output heat flow \(Q_C\)), as shown in Fig. 1, are expressed respectively by Fourier’s equations [20]–[22]:

\[
Q_H = \frac{T_H - T_C}{\theta_m} + \alpha T_H I - \frac{1}{2} R_E I^2
\]
(4)

\[
Q_C = \frac{T_H - T_C}{\theta_m} + \alpha T_C I + \frac{1}{2} R_E I^2
\]
(5)

By observing the electro-thermal model presented in Fig. 3 and applying the current-voltage analogy to the thermal part of the system where current and voltage correspond to heat flow and temperature respectively, the expression of the heat flow through the metal contacts on the hot and cold sides of the TEM can be formulated as:

\[
Q_H = \frac{T_H' - T_H}{\theta_c}
\]
(6)

\[
Q_C = \frac{T_C - T_C'}{\theta_c}
\]
(7)

The difference between the heat flows \(Q_H\) and \(Q_C\) provides the relationship between the heat flow difference and the power transmitted to the load \(R_L\) [16]:

\[
Q_H - Q_C = \frac{T_H' - T_H}{\theta_c} - \frac{T_C - T_C'}{\theta_c} = V_O I
\]
(8)

where \(V_O\) is the output voltage across the load \(R_L\) under constant temperature gradient conditions, and can be defined by:

\[
V_O \bigg|_{\Delta T' = \text{cnst}} = \alpha \Delta T \bigg|_{\Delta T' = \text{cnst}} - R_E I
\]
(9)

Knowing that \(T_H = \Delta T + T_C\), the temperature applied at the cold side of the p and n elements is given by:

\[
T_C = -\theta_c V_O I + \Delta T - \frac{T_H' - T_C'}{2}
\]
(10)

By replacing the temperature \(T_C\) by its expression in Eq. 8 the expression of the thermal gradient at the terminals of the p and n elements with contact thermal resistances can be expressed by:

\[
\Delta T \bigg|_{\Delta T' = \text{cnst}} = \left(\frac{\alpha \theta_c}{\theta_m} + \frac{1}{\theta_m + 2 \theta_c}\right) \times \{\theta_c \theta_m \left[ (\alpha \theta_c V_O + R_E)^2 + (V_O - \alpha (T_H' + T_C')) I \right] + \theta_m \Delta T' \}
\]
(11)

By substituting Eq. 9 in Eq. 10, the thermal gradient at the terminals of the p and n elements under constant temperature gradient conditions becomes:

\[
\Delta T \bigg|_{\Delta T' = \text{cnst}} = \left(\frac{\alpha \theta_c}{\theta_m} + \frac{1}{\theta_m + 2 \theta_c}\right) \times \{\alpha R_E \theta_c \theta_m I^2 + \alpha \theta_c \theta_m (T_H' + T_C') I - \theta_m \Delta T' \}
\]
(12)

By observing Eq. 12 it can be noticed that the temperature gradient \(\Delta T\) seen by the p and n elements of the TEM is load current dependent. Thus, to identify it during the TEM functioning, not only the TEM’s parameters should be known but also the connected load conditions. To simplify the \(\Delta T\) measurement, and consequently the open circuit voltage generated by the TEM, open circuit conditions are usually chosen where the load \(R_L\) is disconnected from the TEM \((I = 0)\), thus bringing Eq. 12 to a much simpler form of temperature divider.

The open circuit voltage generated by the TEM with contact thermal resistances under constant temperature gradient conditions is therefore given by:

\[
V_{OC} \bigg|_{\Delta T' = \text{cnst}, I=0} = \theta_m \frac{\theta_c \theta_m}{\theta_m + 2 \theta_c} \Delta T'
\]
(13)

Fig. 4 illustrates the open circuit voltage measurement of the TEM under constant temperature gradient conditions.

B. TEM modeling under constant heat flow conditions

Under constant heat flow conditions, the input heat flow \(Q_H\) applied to the hot side and the temperature \(T_C'\) at the cold side of the TEM are maintained constant [23].

The thermal gradient at the terminals of the p and n elements for the constant heat flow conditions can be expressed as:

\[
\Delta T \bigg|_{Q_H = \text{cnst}} = \frac{1}{2} R_E \theta_m I^2 - \alpha \theta_c \theta_m T_C I + \frac{Q_H \theta_m}{1 + \alpha \theta_m I}
\]
(14)
In order to get the expression of the thermal gradient \( \Delta T \) only as a function of \( Q_H \) and \( T_C' \), \( T_H \) should be first expressed as a function of these variables. By observing Fig. 3 from the previous section and taking the same current-voltage analogy, the input heat flow \( Q_H \) can be considered as the current flowing through the thermal resistances \( \theta_m \) and \( \theta_c \) connected in series. Thus, by using Eq. 9 the following expression can be obtained for \( T_H' \):

\[
T_H' = T_H + Q_H\theta_c = \Delta T + T_C + Q_H\theta_c
\]  
(16)

Similarly, using Eq. 8 we can write:

\[
Q_H - \frac{T_C - T_C'}{\theta_c} = V_O I
\]  
(17)

which gives:

\[
T_C = -\theta_c V_O I + T_C' + Q_H\theta_c
\]  
(18)

By substituting Eq. 18 in Eq. 16 and then the obtained result in Eq. 15 the thermal gradient \( \Delta T' \) at the terminals of the p and n elements with contact thermal resistances can be expressed with:

\[
\Delta T \bigg|_{Q_H=cnst} = \left( \frac{1}{\alpha \theta_m \theta_c} \right) \times \{ \theta_m (\frac{1}{2} R_E + \alpha \theta_c V_O) I^2 - \alpha \theta_m (T_C' + Q_H \theta_c) I \\
+ Q_H \theta_m \} \tag{19}
\]

Knowing that the output voltage of the TEM under constant heat flow conditions is given by:

\[
V_O \bigg|_{Q_H=cnst} = \alpha \Delta T \bigg|_{Q_H=cnst} - R_E I \tag{20}
\]

and by replacing Eq. 20 in Eq. 19 the expression of the thermal gradient at the terminals of the p and n elements under constant heat flow conditions becomes:

\[
\Delta T \bigg|_{Q_H=cnst} = \left( \frac{1}{\alpha^2 \theta_c \theta_m I^2 - \alpha \theta_m I - 1} \right) \times (\alpha \theta_c \theta_m R_E I^3 - \frac{1}{2} \theta_m R_E I^2 + \alpha \theta_m (T_C' + Q_H \theta_c)I - Q_H \theta_m) \tag{21}
\]

As it has been noticed for the constant temperature gradients conditions, the temperature gradient \( \Delta T \) of the TEM under constant heat flow conditions is also load current dependent. To eliminate this load current dependence, only the open circuit conditions \( (I = 0) \) will be considered during the measurements.

Under constant heat flow conditions, the TEM can be also modeled by an equivalent constant voltage source \( V_O \) in series with an equivalent internal resistance. Here, the open circuit voltage of the TEM is:

\[
V_O \bigg|_{Q_H=cnst, I=0} = \alpha \theta_m Q_H \tag{22}
\]

Fig. 5 illustrates the open circuit voltage measurement of the TEM under constant heat flow conditions.

Based on Eqs. 13 and 22 giving the expressions of the open circuit voltages under both constant temperature gradient and constant heat flow conditions, one can identify the thermal resistances of the TEM only by measuring these two values, as described hereafter in the next section.

III. IDENTIFICATION OF THERMAL RESISTANCES BY ELECTRICAL CHARACTERIZATION

The theoretical thermal resistance \( \theta_m \) of the TEM could be calculated from the following data, if they are provided by the manufacturer:

- \( \rho_{mp} \) the thermal resistivity of the p element.
- \( \rho_{mn} \) the thermal resistivity of the n element.
- \( N \) is the number of pairs of semiconductors of the TEM.
- \( A_p \) and \( A_n \) are the cross sectional areas of p and n elements, respectively.

\[ V_O \bigg|_{Q_H=cnst} = \alpha \Delta T \bigg|_{Q_H=cnst} - R_E I \]

\[ \Delta T \bigg|_{Q_H=cnst} = \left( \frac{1}{\alpha^2 \theta_c \theta_m I^2 - \alpha \theta_m I - 1} \right) \times (\alpha \theta_c \theta_m R_E I^3 - \frac{1}{2} \theta_m R_E I^2 + \alpha \theta_m (T_C' + Q_H \theta_c)I - Q_H \theta_m) \]

\[ V_O \bigg|_{Q_H=cnst, I=0} = \alpha \theta_m Q_H \]
• $L_p$ and $L_n$ are the length of p and n elements, respectively.

The analytical expression of $\theta_m$ is given by [24]:

$$\theta_m = \frac{\rho_m L_p \rho_m L_n}{N (\rho_m L_p A_n + \rho_m L_n A_p)}.$$  \hspace{1cm} (23)

Notice that these data values are not always provided by the manufacturer. Nevertheless, in any case, the thermal resistance $\theta_m$ can be identified using electrical characterization. Based on the electrical model of the TEM under constant heat flow conditions, the open circuit voltage measurement allows us to find the value of $\theta_m$ by knowing the values of the Seebeck coefficient $\alpha$ and the input heat flow $Q_H$, and by using Eq. [24],

$$\theta_m = \frac{V_{oc}}{\alpha Q_H}.$$  \hspace{1cm} (24)

The value of $\alpha$ directly depends on the materials used for the TEM manufacturing and is given by:

$$\alpha = N (\alpha_p - \alpha_n).$$  \hspace{1cm} (25)

where, $\alpha_p$ and $\alpha_n$ are the Seebeck coefficients of the p and n elements, respectively. One can notice that the values of $\alpha_p$ and $\alpha_n$ are mostly provided by the manufacturer.

In order to identify the contact thermal resistance $\theta_c$, the open circuit voltage of the TEM under constant temperature gradient conditions must be measured. This allows us to determine the value of $\theta_c$ by substituting the value of $\theta_m$ previously determined (Eq. [24]), and by knowing the value of the applied thermal gradient $\Delta T'$ (test conditions).

$$\theta_c = \frac{(\alpha \Delta T' - V_{oc})}{2V_{oc}} \bigg|_{\Delta T' = \text{cnst}} \theta_m.$$  \hspace{1cm} (26)

This approach of identifying the internal and contact thermal resistances of a TEM is summarized in Fig. 6. It is based on two open circuit voltage measurements, under both constant temperature gradient and constant heat flow conditions, respectively. The proposed method appears as an efficient and simple way for the TEM’s thermal resistances identification.

IV. CONCLUSION

The Seebeck coefficient $\alpha$ of a TEM is generally provided by the manufacturer; in some cases, the $\theta_m$ value can also be calculated from the provided data. However, the value of the contact thermal resistance $\theta_c$ is not provided while it is required to electrically model the TEM and evaluate its efficiency. In this paper, the identification of the thermal resistances of a thermoelectric module is presented. To do this, some electrical characterizations are necessary. First, the measurement of the open circuit voltage of the TEM, under constant heat flow conditions, allows calculating the value of the internal thermal resistance $\theta_m$. Second, the open circuit voltage of the TEM under constant temperature gradient conditions is measured. This measurement allows to calculate the value of the contact thermal resistance $\theta_c$. Notice that if the $\theta_m$ value is provided by the manufacturer or can be calculated based on the values of $\alpha_p$ and $\alpha_n$, a single electrical test in open circuit and under constant temperature gradient conditions is needed. Moreover, in addition to the parameter identification, the equations of the electrical model of the TEM are given whatever the operating conditions and by taking into account the internal and contact thermal resistances ($\theta_m$ and $\theta_c$).

REFERENCES


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