Novel Flip-OFDM Modulation Scheme based on Discrete Hartley Transform

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***Abstract*—In the domain of visible light communication (VLC), rapid data transmission is based on intensity modulation/direct detection (IM/DD) using flip-orthogonal frequency division multiplexing (flip-OFDM), which is based on the discrete Fourier transform (DFT). Because DFT operates on complex-valued signals, it requires Hermitian symmetry (HS) to produce a real-valued output, thereby doubling computational complexity. This study presents a novel approach that doubles spectral efficiency while halving computational complexity by using the discrete Hartley transform (DHT) instead of the DFT. The analytical results show that the system reduces computational complexity by 55% compared to the conventional scheme. Furthermore, the simulation results show that the new system achieves the same bit-error rate (BER) while reducing the peak-to-average power ratio (PAPR) by 0.3 dB over an additive white Gaussian noise (AWGN) channel and maintains the same performance on a multi-path AWGN channel. The comparison seeks to substantiate the advantages and efficacy of the new system relative to the current technique.**

***Index Terms*—VLC, DHT, flip-OFDM, computational complexity, spectral efficiency, BER, PAPR.**

I. Introduction

ONE traditional multicarrier modulation (MCM) technique used in optical wireless communications (OWC) is orthogonal frequency-division multiplexing (OFDM), which enables high-rate data transfer while alleviating inter-symbol interference (ISI). In optical wireless networks, the intensity modulation/direct detection (IM/DD) technique has been extensively studied for short-distance, inexpensive, high-data-rate optical communications [1]–[5]. Over the past 20 years, wireless communication has advanced significantly. OWC enhances affordable IM/DD solutions for wireless data transfer. The optical wireless communication (OWC) system converts electrical power into a visible-light signal using light-emitting diodes (LEDs) at the transmitter. Photo-diodes (PDs) located at the receiving side convert the strength of the optical signal into an electrical signal at the same time [6], [7]. OWC offers merits such as reduced energy consumption, improved security, higher bandwidth (up to Terahertz), and reduced electromagnetic interference [8]. OWC can take several forms, including visible light communication (VLC). This type of light falls within the frequency range of 430-790 THz, corresponding to a wavelength spectrum of 380-750 nm. VLC effectively addresses issues in radio frequency communication, including interference, limited bandwidth, security concerns, potential health hazards, and suboptimal energy efficiency [9]. The rapid advances in high-power LEDs enable the growth of high-speed VLC networks. The increased peak-to-average power ratio (PAPR) induced by multiple subcarriers in OFDM modulation in VLC systems is a notable downside. This can lead to severe distortion caused by LED nonlinearity [1]. Analogous to DFT-based O-OFDM using 4-256 quadrature amplitude modulation (QAM) constellations, processing with DHT using binary phase-shifted keying (BPSK) to 16-pulse amplitude modulation (16-PAM) can achieve performance levels comparable to those of optical OFDM simulations using DFT. The system's performance has improved relative to previous versions, and Hermitian Symmetry (HS) is no longer required [10], [13]. This study delves into a novel approach. Here is the outline of the paper: an OFDM system that uses the DFT, as described in Section 2. The suggested flip-OFDM technique for VLC systems is described in Section 3. In Section 4, we present the specifications of the proposed system. The analysis and simulation results are presented in Section 5. There is a conclusion at the end of Section 6.

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II. Flip-OFDM scheme based on DFT

In intensity-modulated optical communication systems, the signals used in optical orthogonal frequency-division multiplexing must be real and non-negative. To transmit data, optical communications rely on the brightness of an optical signal, while conventional (non-optical) communications use an electric field. Therefore, the only possible values for the unipolar signal are positive ones [5], [14]. A variety of methods have been used to create a signal with only real positive values, including asymmetrically clipped optical (ACO), DC-biased optical (DCO), and Flip-OFDM [5], [15]. In DCO-OFDM, all subcarriers transmit data, resulting in higher spectral efficiency than in ACO-OFDM. However, according to [14], power efficiency is reduced when a DC bias is applied. Compared with DCO-OFDM, ACO-OFDM techniques are considered more power-efficient, though less efficient in terms of spectral efficiency [15]. The absence of a DC criterion is advantageous. In contrast to other methods, Flip-OFDM introduces a real positive signal without utilizing a DC bias or asymmetric clipping. This increases resistance to distortion caused by clipping and improves power efficiency [16], [17].

III. Proposed flip-OFDM scheme

Figure 1 depicts the suggested VLC system model employing flip-OFDM using the DHT technique analyzed in this work.

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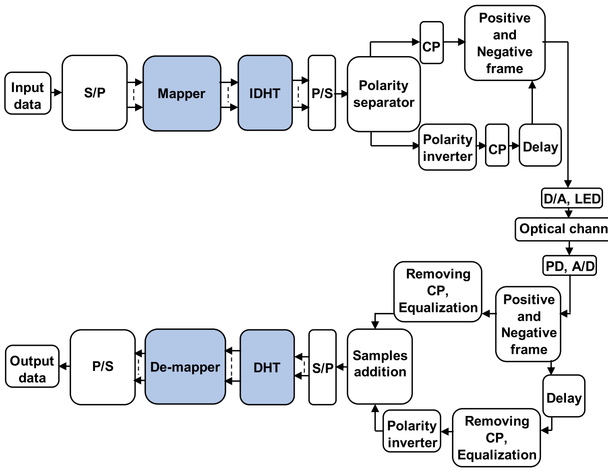

Fig. 1. The block diagram of the proposed Flip-OFDM system based on the DHT.

The data of length N/2 is mapped using M-PAM or BPSK, following the serial/parallel (S/P) conversion, yielding a signal composed exclusively of real values as represented in

$$X_k = (X_0, X_1, X_2, \ldots, X_{N/2-1}) \tag{1}$$

Then the real sequence is processed with the inverse discrete Hartley transform (IDHT) as in,

$$x_n = \frac{1}{\sqrt{N/2}} \sum_{k=0}^{(N/2)-1} X_k \; cas(\frac{2\pi kn}{N/2}) \tag{2}$$

where $cas(.) = cos(.) + sin(.)$ is the DHT kernel [3], and $n = 0, 1, 2, \ldots, N/2-1$ instead of $n = 0, 1, 2, \ldots, N-1$ in the conventional Flip-OFDM based on DFT. That reduces the computational complexity by reducing the number of calculations. The result of the IDHT is real but polar data, which needs to be unipolar, so after the parallel/serial (P/S) conversion, the positive values are separated from the negative values, as in

$$x_n^+ = \max\big(x_n,\, 0\big) \tag{3}$$

$$x_n^- = \max\big(-x_n,\, 0\big) \tag{4}$$

The first equation contains only positive values, while the negative values are set to zero. The second values are negative, while the positive values are set to zero. After adding the cyclic prefix (CP), the two adjacent frames were transmitted via the LED optical channel and received by the PD.

$$y_t = Rx_t \otimes h_t + Z_t \tag{5}$$

Let R represent the responsivity of the photodiode, x_t express the values of the time-domain signal, $\otimes$ indicate a circular convolution, h_t symbolize the channel's response time, and Z_t represent the additive white Gaussian noise (AWGN) integrated into the data signal. On the receiver side, all operations performed by the transmitter are reversed to retrieve the original data: analog-to-digital (A/D) conversion, preparing the positive and negative signals, removing the CP, and followed by the equalization process, then adding the two signals, as in

$$y_n = y_n^+ - y_n^- \tag{6}$$

Sequentially, S/P, DHT, de-mapping, and finally P/s were performed.

IV. Properties Analysis of the Proposed Flip-OFDM System

A. Computational Complexity

The computational complexity (CC) is defined as the number of multiplications (Ms) required by the DFT/IDFT or DHT/IDHT procedures [7]. The computational cost of each N-Point DFT or IDFT, as well as each N-Point DHT or IDHT, is expressed as $N(N\ log_2 N)$ [6]. The computational complexity of the Flip-OFDM system utilizing the DFT or DHT can be expressed as $2N(N\ log_2 N)$ [3], [6]. Utilizing a single N-point IDFT or N-point IDHT in the transmitter, along with a single N-point DFT or N-point DHT in the receiver. The computational complexity of the suggested Flip-OFDM system, using $N/2$-point IDFT/DFT, is $2N(N/2\ log_2 N/2)$. Compared with the traditional DFT system, the computational complexity decreases by more than 55% for N = 1024. Table I presents the computational complexity of the proposed system compared with the existing system for different OFDM frame lengths.

TABLE I
THE COMPUTATIONAL COMPLEXITY (CC) COMPARISONS FOR THE PROPOSED OPTICAL FLIP-OFDM SYSTEM AND THE CONVENTIONAL SYSTEM BASED ON DFT

N	64	256	512	1024
CC Flip-OFDM$_{DFT}$	768	4096	9216	20480
CC Proposed Flip-OFDM$_{DHT}$	320	1792	4096	9216

B. Spectral efficiency

The throughput of each sent OFDM signal denotes the data rate ratio (DRR), defined as the ratio of the original input data to the total transmitted data, and is denoted by R [6]. The data rate reduction (DRR) of the typical flip-OFDM utilizing DFT is denoted as $R_{flip-DFT}$, which is 25% attributable to the implementation of Hermitian symmetry and using positive and negative signals. The spectral efficiency (SE) is expressed as $SE_{flip-DFT} = 1/4\ log_2 M$ (bits/s/Hz), where M represents the modulation order [5]. The data rate reduction (DRR) of the proposed flip-OFDM system, denoted as $R_{flip-DHT}$, is 50%. The spectral efficiency is defined by the equation $SE_{flip-DHT} = 1/2\ log_2 M$ (bits/s/Hz). Table II presents the spectral efficiency of the proposed DHT-based flip-OFDM system, compared with the existing system across different modulation orders.

C. PAPR analysis

When the IDHT process is given by (2), assuming the X_k are an Independent, identically distributed (i.i.d.), zero mean,

TABLE II
THE SPECTRAL EFFICIENCY COMPARISONS FOR THE PROPOSED OPTICAL FLIP-OFDM SYSTEM AND THE CONVENTIONAL SYSTEM BASED ON DFT

M	64	256	512	1024
CC Flip-OFDM$_{DFT}$	1.5	2	2.25	2.5
CC Proposed Flip-OFDM$_{DHT}$	3	4	4.5	5

with variance $Var[X_k] = \sigma_X^2$, and due to the orthogonality of the DHT matrix, therefore, every time domain signal x_n has same variance, $Var[x_n] = \sigma_X^2$, and $E[x_n] = 0$, using

$$\frac{1}{N}\sum_{k=0}^{N-1} \text{cas}^2\left(\frac{2\pi kn}{N}\right) = 1 \tag{7}$$

The Gaussian approximation (Central Limit Theorem)

$$x_n \overset{\text{approx}}{\sim} \mathcal{N}(0, \sigma_X^2) \tag{8}$$

For a large number of N. The PAPR expression for each OFDM block is given by

$$\text{PAPR} = \frac{\max_{0\le n<N} x_n^2}{E[x_n^2]} = \frac{\max_n x_n^2}{\sigma_X^2} \tag{9}$$

If normalized power as in

$$Z = \frac{x_n^2}{\sigma_X^2} \tag{10}$$

Then

$$\Pr(Z > \gamma) = \Pr(|x_n| > \sigma_X \sqrt{\gamma}) = 2\,Q(\sqrt{\gamma}) \tag{11}$$

Where γ is the PAPR threshold and the Q(.) is the Gaussian Q-function. While for conventional flip-OFDM based on DFT,

$$\Pr(Z > \gamma) \approx e^{-\gamma} \tag{12}$$

The complementary cumulative distribution function (CCDF) is explained as

$$\text{CCDF}_{\max}(\gamma) = \Pr\left(\max_n Z_n > \gamma\right) = 1 - (1 - 2Q(\sqrt{\gamma}))^N \tag{13}$$

Although the CCDF for the conventional flip-OFDM system is

$$\text{CCDF}_{\max}^{\text{DFT}}(\gamma) \approx 1 - \left(1 - e^{-\gamma}\right)^N \tag{14}$$

Since each signal of the flip-OFDM signals (positive and negative) represents a half-wave rectified form of the x_n, for the positive values signal, the probability of being zero is

$$\Pr(x_n \le 0) = \frac{1}{2} \tag{15}$$

so

$$\Pr\left(\frac{(s^{(+)})_n^2}{\sigma_X^2} > \gamma\right) = Q(\sqrt{\gamma}) \tag{16}$$

$$\text{CCDF}_{\max}(\gamma) = 1 - \left(1 - Q(\sqrt{\gamma})\right)^N \tag{17}$$

Similarly, for negative-valued signals, the PAPR of the two proposed DHT-based flip-OFDM signals is roughly equal to that of the conventional DFT-based flip-OFDM system.

$$\Pr\left(\frac{x_n^2}{\sigma_X^2} > \gamma\right) = 2Q(\sqrt{\gamma}) \tag{18}$$

$$\text{CCDF}_{\max}^{\text{FLIP}}(\gamma) = 1 - \left(1 - 2Q(\sqrt{\gamma})\right)^N \tag{19}$$

D. BER analysis

The symbol error probability for P_s for M-PAM is

$$P_s \approx 2\left(1 - \frac{1}{M}\right) Q\left(\frac{d}{2\sigma_n}\right) \tag{20}$$

Where d is the minimum distance between nearby PAM levels and can be expressed by

$$d = \sqrt{\frac{12E_s}{M^2 - 1}} \tag{21}$$

Where E_s is the symbol energy and the symbol spacing, which is

$$E_s = \frac{d^2}{12}(M^2 - 1) \tag{22}$$

By substituting d in the BER formula

$$P_s \approx 2\left(1 - \frac{1}{M}\right) Q\left(\sqrt{\frac{3|H_k|^2 E_s}{(M^2 - 1)N_0}}\right) \tag{23}$$

Where N_0 represents the noise power spectral density and the H_k is the channel response in the frequency domain. From the analysis above, the BER of the proposed flip-OFDM system based on DHT is analytically equivalent to that of the traditional flip-OFDM system based on DFT.

V. Simulation Results

The efficacy of the proposed method is evaluated with respect to peak-to-average power ratio (PAPR) reduction and bit error rate (BER) performance. The R2021b MATLAB software is used to execute simulations, and Table 3 presents the parameters evaluated for these simulations.

TABLE III
Simulation Parameters

Parameters	Specifications
System	VLC system based on DHT-Flip-OFDM
Number of frames	10000
Modulation	32-PAM and 1024-PAM
Spectral efficiency	2.5 (bits/s/Hz)
OWC channel	Multi-path and AWGN
Tap delays	[0, 2, 5]
Tap weights	[1, 0.4, 0.15]
Equalizer	Regularized zero-forcing (RZF)
CP length	N/4

VLC systems exhibit varying spectral efficiencies at distinct modulation orders. Simulations and analyses were conducted at a specific SE for a fair comparison. For instance, 1024-QAM is used with flip-OFDM, while 32-PAM is used with the new flip-OFDM scheme based on DHT to achieve 2.5 (bits/s/Hz) spectral efficiency. Fig. 2 illustrates the CCDF performance for the conventional flip-OFDM scheme based on DFT and the proposed Flip-OFDM based on DHT.

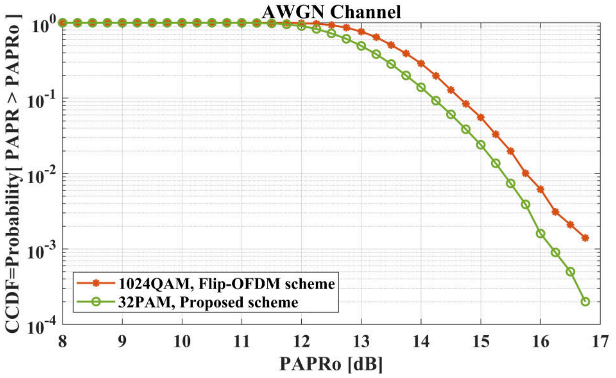

Fig. 2. CCDF performance of the conventional flip-OFDM with 1024-QAM and the proposed flip-OFDM based on DHT with 32-PAM, where the SE is 2.5 (bits/s/Hz) using an AWGN channel.

The results show that the new system reduces PAPR by 0.3 dB relative to DFT-based flip-OFDM at $CCDF = 10^{-2}$.

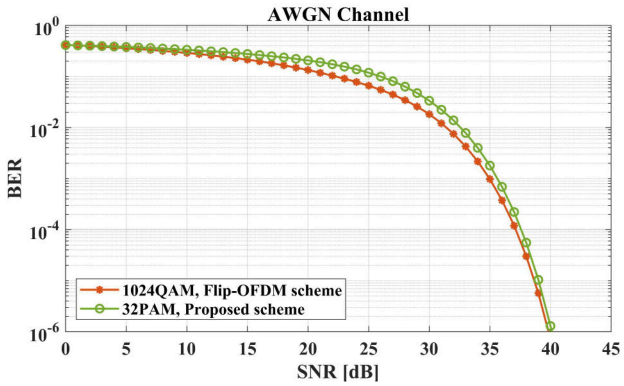

Fig. 3. BER performance vs SNR of the conventional flip-OFDM with 1024-QAM and the proposed flip-OFDM based on DHT with 32-PAM, where the SE is 2.5 (bits/s/Hz) using an AWGN channel.

Moreover, the BER performance is shown in Fig. 3. The results indicate that the proposed scheme achieves performance comparable to that of the traditional scheme at $BER = 10^{-6}$.

For greater reliability, the optical wireless transmission channel is modeled as a real-valued multi-path VLC channel followed by an AWGN channel. The multi-path channel dispersion is modeled using a discrete-time tapped-delay line with tap delays [0, 2, 5] and tap weights [1, 0.4, 0.15]. Noise is added independently to each OFDM frame of the generated Flip-OFDM signals, according to the traditional AWGN model. Fig. 4 shows the CCDF performance for the proposed flip-OFDM system compared to the conventional flip-OFDM at the same spectral efficiency 2.5 (bits/s/Hz) and the same number of transmitted sub-carriers N=1024.

The proposed system exhibits the same CCDF performance as the conventional system at $CCDF = 10^{-2}$. Fig. 5 presents the BER performance of the two systems under the same

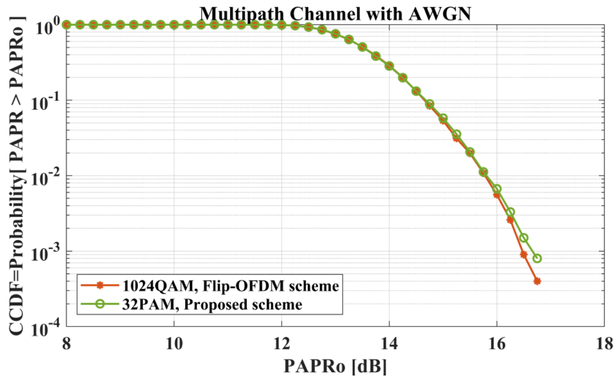

Fig. 4. CCDF performance of the conventional flip-OFDM with 1024-QAM and the proposed flip-OFDM based on DHT with 32-PAM, where the SE is 2.5 (bits/s/Hz) using a multi-path channel with AWGN.

conditions, namely, a multi-path and an AWGN channel, with the same spectral efficiency of 2.5 (bits/s/Hz).

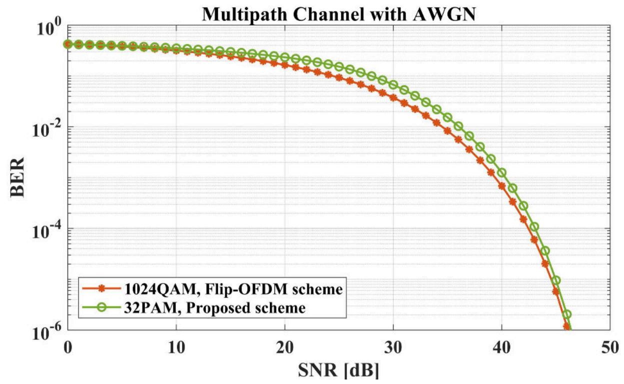

Fig. 5. BER performance vs SNR of the conventional flip-OFDM with 1024-QAM and the proposed flip-OFDM based on DHT with 32-PAM, where the SE is 2.5 (bits/s/Hz) using a multi-path channel with AWGN.

The simulation results show nearly identical BER performance for the proposed flip-OFDM scheme compared with the standard flip-OFDM based on DFT, indicating no performance degradation from using DHT instead of DFT in the flip-OFDM modulation scheme.

VI. CONCLUSION

In this article, a flip-OFDM system based on DHT is proposed. It has the advantage of not requiring Hermitian symmetry. The new flip-OFDM system had the lowest computational complexity of all, at 4096 Ms for $N = 1024$, compared with 20480 Ms for the conventional flip-OFDM system based on DFT. Regarding spectral efficiency, the new system achieved the highest, 5 bits/s/Hz, at a modulation order of 1024, whereas the traditional system achieved 2.5 bits/s/Hz. Furthermore, the new system showed a significant improvement in PAPR of approximately 0.3 dB, with no degradation in BER on an AWGN channel, while achieving the same performance as the conventional DFT-based flip-OFDM on DFT using a multi-path channel. In future work, we may use advanced techniques, such as precoding or spatial modulation, to mitigate the high PAPR and improve overall system performance.

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